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Alexey A. Melnikov, and Leonid E. Fedichkin

Fermionic entanglement via quantum walks in quantum dots

Alexey A. Melnikov[1,a),b)] and Leonid E. Fedichkin[1,2,3,c)]

[1]*Institute of Physics and Technology, Russian Academy of Sciences, Nakhimovskii pr. 36/1, Moscow 117218, Russia*
[2]*NIX, Zvezdny boul. 19, Moscow 129085, Russia*
[3]*Moscow Institute of Physics and Technology, Institutskii per. 9, Dolgoprudny 141700, Russia*

[a)]Corresponding author: melnikov@phystech.edu
[b)]URL: melnikov.info
[c)]leonid@phystech.edu

Abstract. Quantum walks are fundamentally different from random walks due to the quantum superposition property of quantum objects. Quantum walk process was found to be very useful for quantum information and quantum computation applications. In this paper we demonstrate how to use quantum walks as a tool to generate high-dimensional two-particle fermionic entanglement. The generated entanglement can survive longer in the presence of depolorazing noise due to the periodicity of quantum walk dynamics. The possibility to create two distinguishable qudits in a system of tunnel-coupled semiconductor quantum dots is discussed.

INTRODUCTION

Quantum walks [1], which are quantum analogues of random walks, have been used to develop new tools for quantum information theory such as e.g. new quantum search algorithms [2]. Quantum walks have also been proven to be capable of universal quantum computation both in a single-particle [3] and a multiparticle [4] scheme. Such schemes could be used to create a scalable quantum computer without a need for time-dependent control. It is hence important to study fundamental properties of quantum walk dynamics on graphs, also by considering feasible experimental implementations [5].

Here we study properties of quantum walk dynamics on a cycle graph. This graph consists of vertices $n \in \{1 \dots N\}$ and edges e from the set $E \in \{(1,2),(2,1),\dots,(N,1),(1,N)\}$ as schematically depicted in Fig. 1. The quantum walk evolution on the cycle graph is governed by the following XY Hamiltonian:

$$H = \hbar \frac{J}{2} \sum_{i,j \in E}^{N} \left(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right), \tag{1}$$

where σ^x and σ^y are the Pauli matrices acting on an excitation degree of freedom of a vertex, J is a coupling strength. A particle (an excitation) that starts from a certain vertex will continuously evolve to a superposition of being in nearest-neighbor sites. This superposition makes a fundamental difference between quantum and classical particles, which is visible from the transport properties.

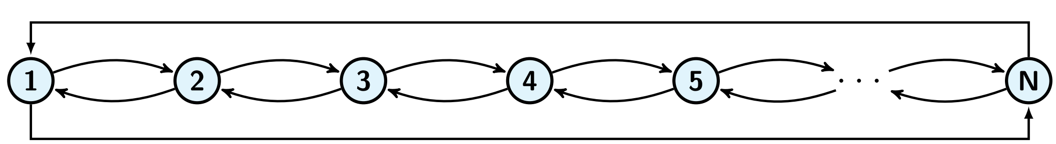

FIGURE 1. Schematic representation of a cycle graph as a line with boundary conditions. All edges correspond to the possible transitions that happen with the same probability.

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In Fig. 2 one can see that the quantum particle traverse the cycle graph faster than its classical analogue. The distance, which is defined as the minimum number of edges that separates the particle from its initial position, increases during the considered time for both quantum and classical particle. However, as one can see in this example, for the quantum particle the distance increases linearly while for the classical particle the distance grows only quadratically.

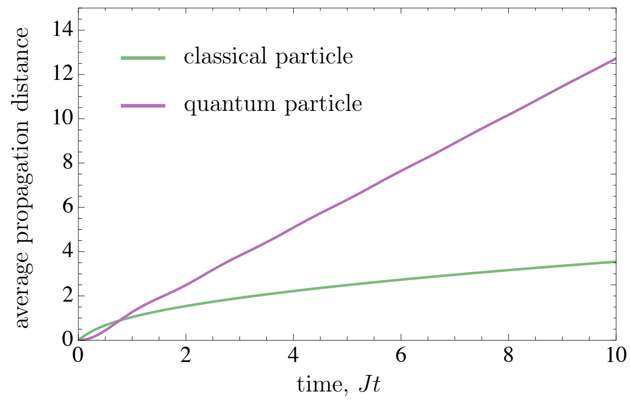

FIGURE 2. The average propagation distance of a classical and a quantum particle as a function of time. A quantum walk is performed on a cycle graph with 50 vertices.

Semiconductor quantum dots for quantum walks

As a feasible physical implementation of a quantum walk process, one can consider an array of tunnel-coupled semiconductor quantum dots that are arranged in a circle. Experimentally, lateral structures of this geometry with different number of quantum dots are readily implementable. A concept of a scalable architecture was demonstrated by fabricating quadruple and quintuple quantum dots [6, 7]. Similar techniques and technologies could be used for a fabrication of circles of larger sizes.

If one leaves only one electron in this quantum system, then this electron will walk and spread coherently showing one-particle continuous-time quantum walk dynamics [8]. In addition to a possibility to observe the quadratic speed-up in propagation time shown in Fig. 2, one can define a qubit in this semiconductor system. Semiconductor charge qubits can be protected from errors [9, 10] by using standard quantum error correction algorithms [11, 12], or by means of encoding the qubits in a decoherence-free subspace [13].

RESULTS

By injecting the second electron into the system of quantum dots, a richer dynamics of charge distribution over the cycle can be observed due to mutual Coulomb repulsion between two electrons. But apart from the charge distribution dynamics we show that quantum walk of two identical electrons in the considered system gives rise to fermionic entanglement [14, 15]. Entanglement plays a pivotal role in most of the branches of quantum information science, but the use of identical sub-subsystems (such as electrons in quantum dots) introduces additional challenges. An origin of these challenges is in an antisymmetric description of fermionic wave function of two electrons:

$$|\psi\rangle = \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\alpha_{ij}\,|\psi^{(i,j)}\rangle, \tag{2}$$

$$|\psi^{(i,j)}\rangle = \frac{1}{\sqrt{2}}\Big(|i,j\rangle - |j,i\rangle\Big). \tag{3}$$

The fermionic entanglement of the state $|\psi\rangle$ is defined using the definition of the Slater rank, which is the minimum number of the Slater determinants that describe the fermionic state $|\psi\rangle$. Some examples of using this definition to determine whether a certain fermionic state is entangled are given in e.g. Ref. [16]. Quantifying fermionic entanglement remains a theoretical challenge, but progress is made in defining concurrence in systems of identical fermions [17].

Let us consider the evolution of two electrons in quantum dots, modelled by the single-particle quantum walk Hamiltonian from Eq. 1 with additional repulsive interaction between two particles H_{int} [14]. The evolution of the fermionic density matrix in a presence of noisy environment is

$$\rho(t) = e^{-\gamma t} e^{-iH_{\text{int}}t/\hbar} \, |\psi(0)\rangle \langle \psi(0)| \, e^{iH_{\text{int}}t/\hbar} + \left(1 - e^{-\gamma t}\right) \rho_{\text{mix}}, \tag{4}$$

where ρ_{mix} is the maximally mixed state of two electrons. A presence of a depolarizing noise can lead to dissociation or annihilation of entanglement [18, 19]. In the scenario we consider, the affect of noise on the fermionic state is demonstrated in Fig. 3 for the cycle graph with 6 vertices. Note that this two-particle entanglement of high-dimensional subsystems can survive in a noisy environment due to periodicity of the quantum walk dynamics.

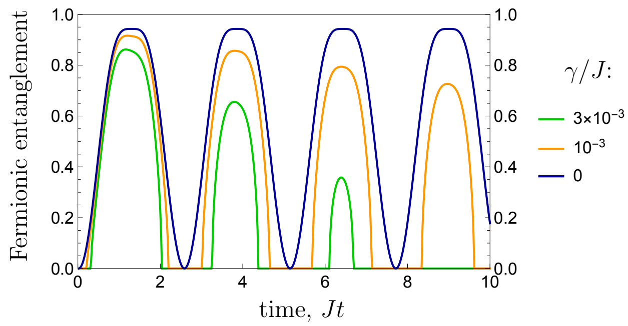

FIGURE 3. Fermionic entanglement dynamics in the presence of noise and without noise (blue).

Fig. 3 shows that during the continuos-time quantum walk process the fermionic entanglement is generated by starting from a separable state of two maximally spatially separated electrons of the same spin. However, it was not apparent how useful are these fermionic entangled states for quantum information processing, because of inconsistency between fermions and qubits: the Hilbert space of two fermions is an antisymmetric product, not a direct product. In Ref. [14] we show how one can define qudits (d-level quantum bits) by using the freedom of dividing the graph into two subgraphs. This quantum walk dynamics of two indistinguishable fermions, in particular, can be used in preparation of two-qudit entanglement of two distinguishable subsystems. The process of generating entangled states of two distinguishable qubits, starting from a separable state of two identical fermions, has also been recently studied in Ref. [20].

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