Charge qubit decoherence caused by effect of image forces

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Once a charge is brought to a metal, the charges on its surface distribute in such a way that not to allow an electric field to penetrate it. That is a well known effect of electromagnetism. The same situation arises when a charge qubit is placed near a metal that often takes place because of gates. Since an electron is moving during calculation the charge on the surface is moving in the same way. But in doing so it indispensably entails a Joule loss in the gate. Thus the energy of a qubit dissipates that results in decoherence in the system. The calculation below shows the effect may be great enough to demolish the qubit state.

I. INTRODUCTION

The charge-based qubits look as quite promising for a solid state quantum computer implementation. However, almost all proposals of qubits of the kind include the gates for their operation. At the same moment, it is well known that gates can cause decoherence in the system, in particular, via the thermal noise in a gate voltage [1]. However, decoherence of the type could be suppressed by low temperature. Here we discuss some more processes regarding the gates which can also encumber the qubit functioning.

Coulomb interaction of an electron with the gates producing image forces can also result in decoherence. The reason is that the moving charge in the qubit creates the moving charge in the gate which indispensably entails a resistivity loss. We had estimated the rate of this loss and recovered that the associated decoherence of the qubit might be substantial. Certainly, the feasible way to avoid it lies in employment of superconducting materials for gate electrodes. Another possibility is to do without gates at all and exploit laser pulses for driving the qubits [2–4].

II. QUANTUM CONSIDERATION

The structure under consideration is sketched in Fig. 1. It consists of a double quantum dot (DQD) placed near a metal surface which indispensably exists in any structures operated by gates. The DQD contains one electron. If an electron were a classical particle, we would deal with a well known task of electrostatics. The solution of this problem is based on image charge introduction. This charge is of opposite sign and is located symmetrically regard to a metal surface. As far as quantum electron is concerned one should resort to quantum mechanics formalism. But it is not clear whether "image charge" approach remains correct. At first sight the problem is not difficult at all. We must only add image force potential $-\frac{e^2}{2r}$ to the confining potential of DQD. The question arises what is the real charge distribution in metal corresponding to the image charge. Thus the obstacle appears how to introduce an image force potential into Schrödinger equation.

The first proposals to solve the problem were made in [5, 6]. Authors suggested surface charge to distribute itself in response to the *external* electron charge in a way which is described classically by the image charge. According to quantum mechanics the *external* charge is given by $e|\psi(\mathbf{r},t)|^2$, where $\mathbf{r} = (x, y, z)$, with z being equal to 0 on metal surface. The metal surface charge responds to this charge distribution. It results in image force potential

$$V_{im}(x,y,z,t) = -\int \frac{e^2|\psi(x',y',z',t)|^2dx'dy'dz'}{\sqrt{(x-x')^2+(y-y')^2+(z+z')^2}}. \tag{1}$$

In Ref. [7] such an image force potential has been used successfully to calculate a perturbation of atomic energy levels caused by metal surface. It can be shown that such a potential is a consequence of Hartree mean-field approach applied to the system "metal - qubit". In this particular case such an approach is acceptable because the characteristic time of electron oscillation in a qubit is much longer than the relaxation time in metal which is of the order of $10^{-13}sec$.

III. DECOHERENCE

Although the imaginary charge is situated deep under the metal surface, really, it is produced by charge displacement quite close to the surface (Fig. 1). The thickness of this layer is about Tomas-Fermi screening length r_{TF} which is about several Ångstrøms in metals. Therefore, the resistivity is determined by surface roughness scattering rather than bulk phonon scattering. It is known that unlike phonon scattering the surface scattering is almost independent of temperature.

When electron is located in one quantum dot its wave function resembles a delta function, i.e. electron can

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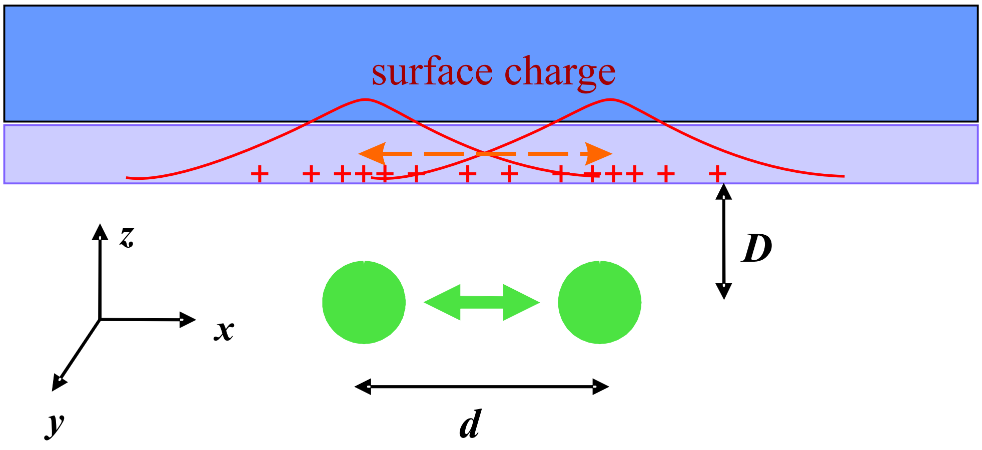

FIG. 1: The moving charge in the qubit drags charges in metal that indispensably entails Joule loss: d is a double dot separation and D is a distance to the metal surface.

be regarded as a point charge. Oscillations of electron in DQD can be treated roughly as transitions of such a charge. The charge density $\rho(x,y,t)$ at the metal surface induced by an point charge moving with the velocity v along the surface in x-direction at a distance D from the surface is supplied by the solution of the relevant electrostatic problem [8]:

$$\rho(x,y,t)=\frac{eD}{2\pi\left((x-vt)^2+y^2+D^2\right)^{3/2}}. \tag{2}$$

The associated surface current density $j(x,y,t)$ obeys the continuity equation

$$\frac{\partial j}{\partial x}=-\frac{\partial \rho}{\partial t} \quad , \tag{3}$$

which gives rise to

$$j(x,y,t)=v\rho(x-vt,y)=\frac{veD}{2\pi\left((x-vt)^2+y^2+D^2\right)^{3/2}}. \tag{4}$$

Joule loss power W_{J} is

$$W_J=\int dx\int dy\frac{j^2(x,y,t)}{\sigma_s}, \tag{5}$$

where $\sigma_s=\sigma r_{\mathrm{TF}}$ is a sheet specific conductivity providing σ is a bulk specific conductivity of metal. Here we assume that the charges are moving inside the layer with thickness r_{TF} under the metal surface. After integration in the equation (5) one arrives at

$$W_J=\frac{v^2e^2}{16\pi\sigma r_{\mathrm{TF}}\cdot D^2}. \tag{6}$$

The Joule dissipation energy lost over the period T of Rabi oscillations of the qubit is

$$E_J\approx\frac{e^2\cdot d^2}{8\pi\cdot T\sigma r_{\mathrm{TF}}\cdot D^2}, \tag{7}$$

where the relation $d=v/T$ was taken into account.

The associated quality Q of Rabi oscillations is

$$Q\approx\frac{h\nu}{E_J}\approx\frac{8\pi h\sigma r_{\mathrm{TF}}\cdot D^2}{e^2\cdot d^2}, \tag{8}$$

where $\nu=1/T$ is the oscillation frequency and h is Plank constant. If qubit is placed perpendicular to a metal surface the energy loss will be lower and quality is about one order greater.

It can not help mentioning that Joule energy loss is insensitive to dielectric permittivity because this constant does not change the quantity of free charge in metal and only affects bound charge in dielectric.

For realistic structures we have substituted $r_{\mathrm{TF}}=5\mathring{A}$, $D=d=10nm$ and $\sigma=10^5(Ohm\ m)^{-1}$ in the relation (8). The specific conductivity σ has been taken from Refs. [9, 10] where the experiments with thin metallic films were presented. There was shown that the dominating surface scattering was insensitive to temperature, therefore, the conventional bulk conductivity rule $\sigma\sim T^{-3}$ was broken down. Moreover, since we regard a surface conductivity the free length of electrons is of the order of lattice spacing and consequently r_{TF} . The substitution of those parameters gives $Q\sim 10$. The obtained magnitude points to the peculiarity of “qubit-gate” decoherence processes and can be a main course of strong relaxation of Rabi oscillations seen in the experiment with doudle quantum dots [11, 12].

IV. CONCLUSIONS

As the moving charge in the qubit creates the moving charge in a gate this indispensably entails a Joule loss in the gate. It is one more cause of decoherence in the system. Unluckily, it cannot be suppressed by low temperature as the charge in metal is moving close to the surface and, therefore, the resistivity is determined by surface roughness scattering rather than by bulk phonon scattering. It is known that the surface scattering is quite insensitive to temperature. The estimations have shown that in realistic gated double dot structures the quality of Rabi oscillations may be less than 10. The employment of superconducting materials for gates instead of normal metals is claimed. A light driven qubit is also a feasible issue. The perfectly symmetrical charge qubit structure where there is no charge transfer is the other way out as well [13, 14].

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