Continuous-time fermionic quantum walks

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Introduction

We study quantum walks of identical particles on graphs. Due to the quantum information perspectives, the interest to quantum walks increased recently [1]. Apart from quantum information applications, quantum walks may explain the energy transfer within photosynthetic systems [2] and provide the speedup to active learning agents [3].

There are several ways to define quantum walks, in this work the walks are continuous-time and defined by the time-independent Hamiltonian

$$H = \Omega \sum_{i=0}^{N-1} |x_{i+1 \bmod N}\rangle\langle x_i| + |x_i\rangle\langle x_{i+1 \bmod N}|,$$

where x_i is the coordinate of the node i with the total number of nodes N, Ω is the tunnelling amplitude. It was shown that one-particle continuous-time quantum walk of the described form could perform any quantum computation and the necessary gates were provided [4].

Quantum walks of electron in quantum dot nanostructures

Quantum dots are promising elements for quantum computations. Quantum dot qudit consists of quantum dots with one electron connected by tunnelling. Dots themselves are formed from the two-dimensional electron gas by field of gates and these dots are controlled by potentials on gates. As a result we have a qudit basis states $|0\rangle$, $|1\rangle$, ... and $|N\rangle$, which describe the localization of an electron in 0-th, 1-st or N-th place, respectively. The ability to perform some quantum operations was proven. The quantum walks of non-interacting distinguishable particles are equivalent to one-particle walks, whose dynamics have already been studied [5].

The probability of particle to be in the node 0 at time t [5] is

$$P_0(t) = \frac{1}{N} + \sum_{m,n=0}^{N-1} \frac{1 - \delta_{m+n,0} - \delta_{m+n,N}}{N^2} \times$$

$$\times \exp\left(4i\Omega t \sin\frac{\pi(m+n)}{N}\cos\frac{\pi(m-n)}{N}\right).$$

For $N = 4$ and $N = 6$ there are the periods of quantum walks. They are $T = \pi/\Omega$ and $T = 2\pi/\sqrt{3}\Omega$, respectively [6].

Two-electron quantum walk

In universal quantum computation by one-particle quantum walk the number of components scales exponentially with the number of encoded qubits. One can decrease the number of components by using two particles in the quantum walk. One way of realizing the two-particle quantum walk is by two electrons tunneling between the quantum dots in silicon.

We consider a cycle graph with N nodes and two identical electrons in these nodes. Hopping Hamiltonian of the non-interacting electrons is

$$H = \Omega \sum_{i,j=0}^{N-1} |x_{i+1}, y_j\rangle\langle x_i, y_j| + |x_i, y_j\rangle\langle x_{i+1}, y_j| +$$
$$+ |x_j, y_{i+1}\rangle\langle x_j, y_i| + |x_j, y_i\rangle\langle x_j, y_{i+1}|,$$

where x_i and y_i are the coordinates of the first and the second indistinguishable particle respectively. Spins of the electrons are directed upwards $|\uparrow\uparrow\rangle$. The graph and the charge density dynamics are shown in Figs. 1 and 2, respectively.

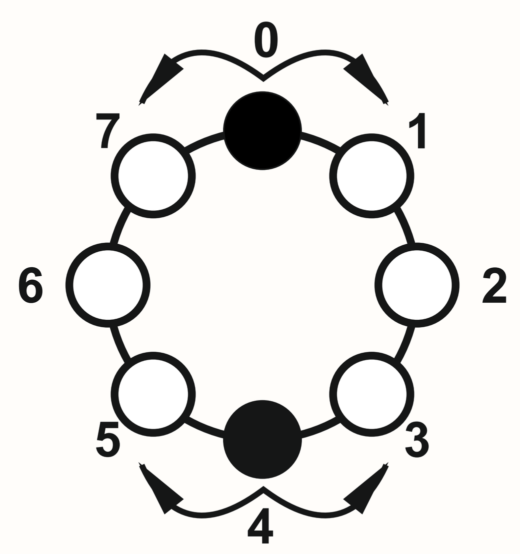

Figure 1: The scheme of two-particle quantum walk on a cycle graph with 8 nodes. Electrons are initially in the 0-th and the 4-th nodes, that is the fermionic state is $|\psi\rangle = (|04\rangle - |40\rangle)|\uparrow\uparrow\rangle/\sqrt{2}$.

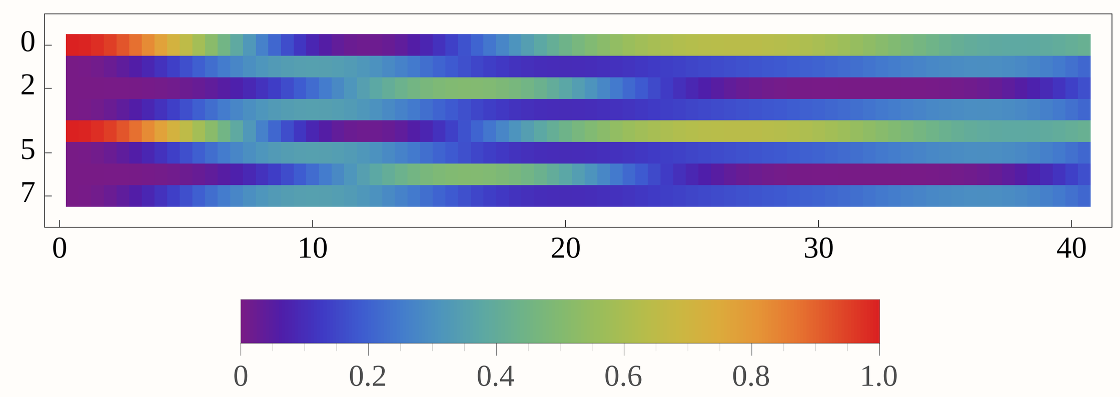

Figure 2: Charge density dynamics in all nodes. There is no hopping period as it is in the cases of 4 and 6 nodes. The graph and the initial state are shown in Fig. 1.

Interacting electrons

The hopping Hamiltonian with the restriction of being in the neighbouring sites of the circle graph because of Coulomb interaction [6]:

$$H = \Omega \sum_{i,j=0;j\neq i,i\pm 1,i+2}^{N-1} |x_{i+1}, y_j\rangle\langle x_i, y_j| +$$
$$+ |x_i, y_j\rangle\langle x_{i+1}, y_j| + |x_j, y_{i+1}\rangle\langle x_j, y_i| + |x_j, y_i\rangle\langle x_j, y_{i+1}|.$$

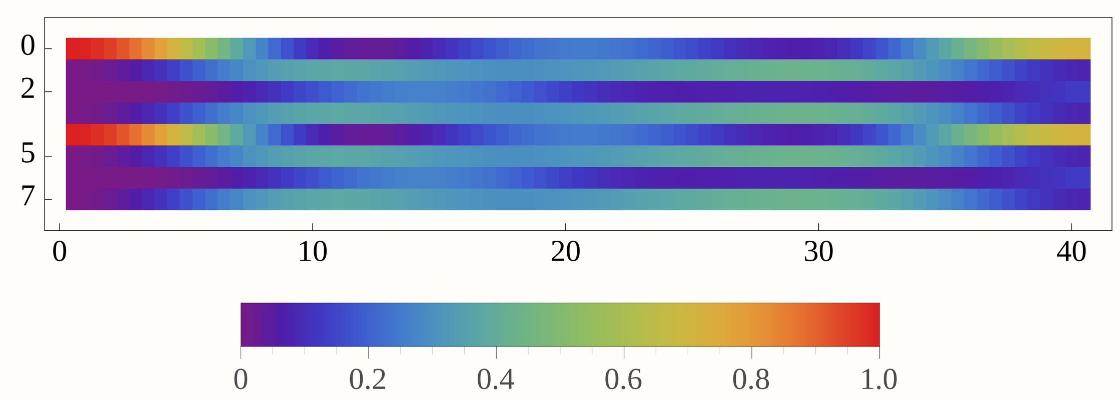

Figure 3: Charge density dynamics in the case of interacting electrons. The graph and the initial state are shown in Fig. 1.

Coulomb interaction causes the quantum correlations between the electrons, i.e. qudits become entangled. This fact can be estimated by a definition of the fermionic entanglement. The fermionic wave function is called entangled if Slater rank is grater than one. Or, equivalently, a pure state of K identical fermions is separable if and only if the purity of the single-particle reduced density matrix is equal to $1/K$. If $1/n \leq \mathrm{Tr}\rho_1^2 < 1/K$, then the Slater rank is grater than one and the state is entangled. n is the dimension of the single particle state space.

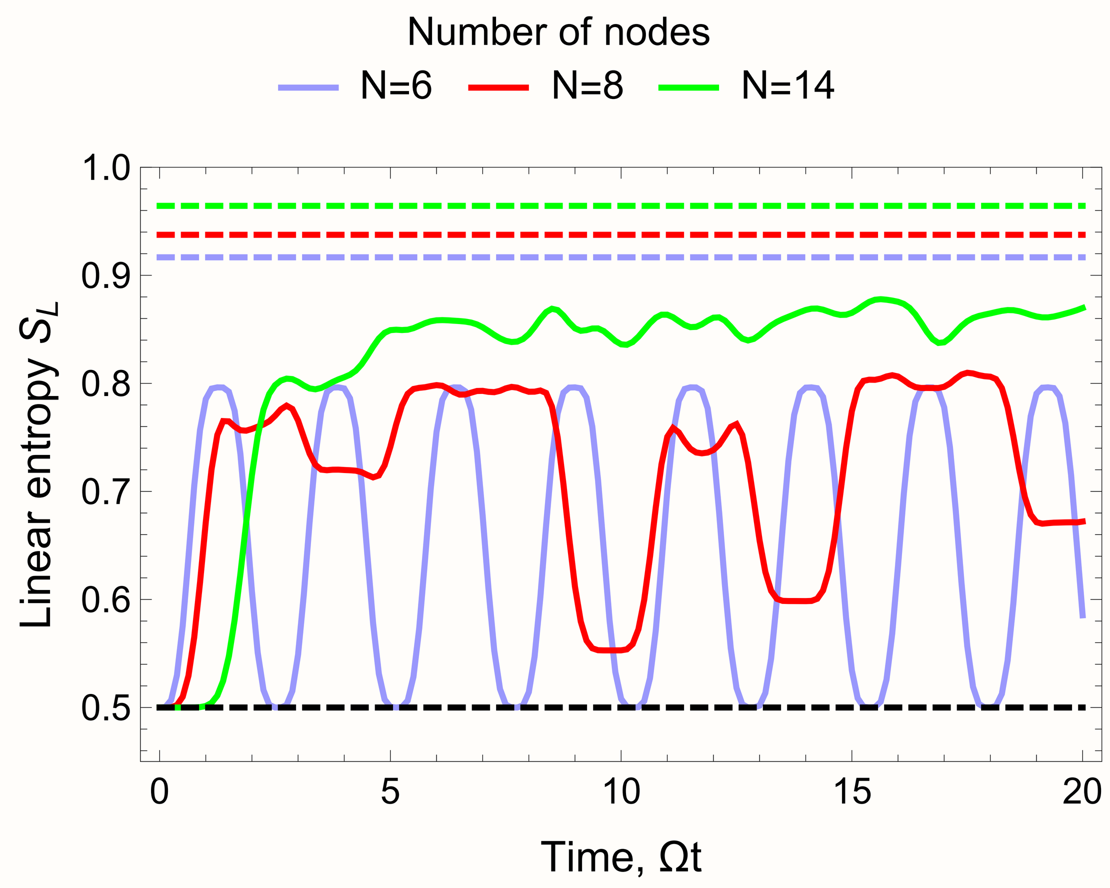

Figure 4: Linear entropy $S_L = 1 - \mathrm{Tr}\rho_1^2$ represents the entanglement dynamics. The dynamics starts from a separable state $(\mathrm{Tr}\rho_1^2 = 1/2)$. One can see that $S_L > 1/2$ for $t > 0$, i.e. two fermions become entangled during the quantum walk. In the case with $N = 6$ nodes system returns periodically to the separable state (black dashed line). The dashed lines of corresponding colors show the upper bound for entangled states [6].

Influence of decoherence on quantum walks

Semiconductor quantum dots are promising elements for quantum computation, but decoherence is a significant obstacle. The decoherence process of a one-particle state in this system was studied in [5] and we study it further by considering two interacting fermions. We investigate the entanglement dynamics in the realistic scenario of decoherence presence. The proposed scheme for two-particle quantum walk could be used for entangling gates construction.

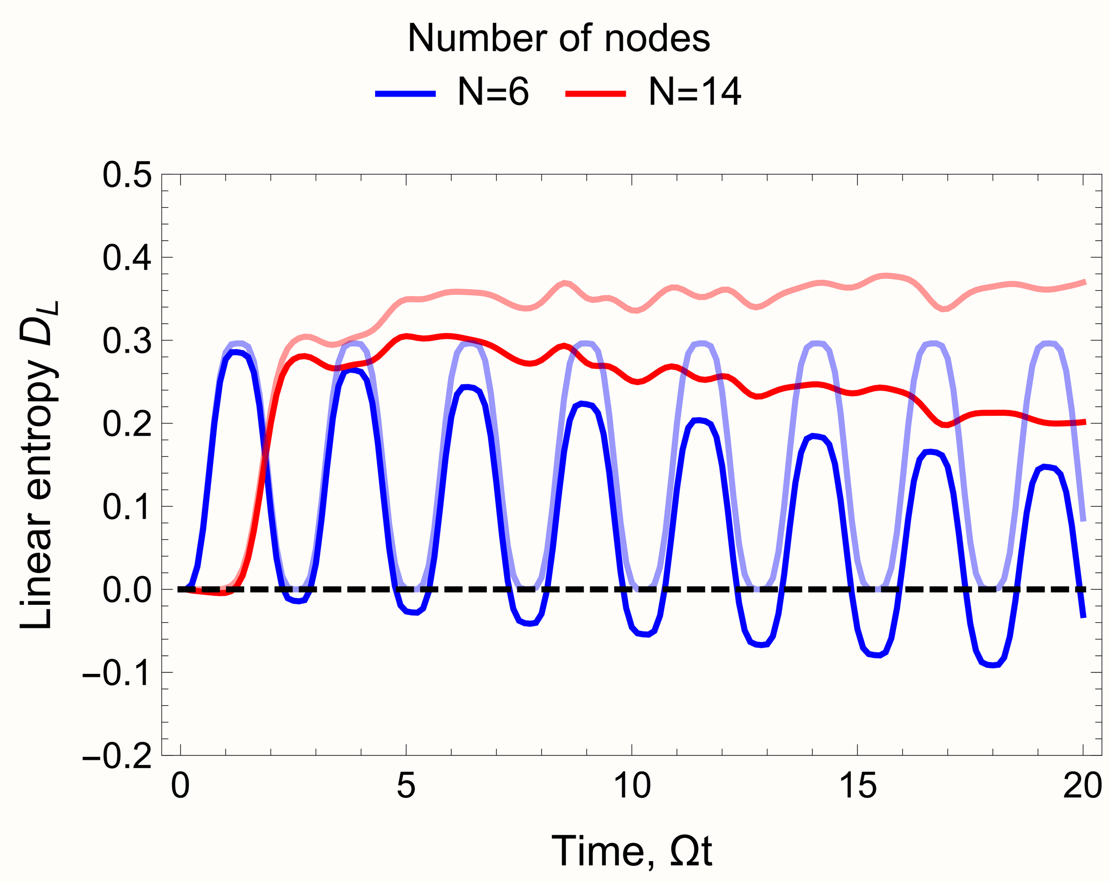

Figure 5: Linear entropy $D_L = S_L(\rho_1) - S_L(\rho) - 1/2$ represents the entanglement dynamics of the mixed fermionic state. In the regions, where $D_L > 0$, electrons are entangled. Depending on the noise parameters fermionic entanglement can be created for a certain time or re-created after annihilation.

References

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